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| 11/12 Breakout: Session #1: Big Ideas of Proportional Reasoning | | Grade 11/12 |
| 90 min | Math Learning Goals  Explore the Big Ideas of Proportional Reasoning  Make connections between the overall (specific?) expectations to the big ideas of Proportional Reasoning  Practice matching problems involving proportional reasoning to the Big Ideas | Materials   * BLM1.1 Connecting Problems to BI * Big Ideas from program * Strips of Curriculum expectations in envelopes by course * BLM1.2 Curriculum Expectations |
|  | Whole Group 🡪 Sharing   * Ice breaker (getting to know each other/response to plenary - beach ball activity). * Establish and post group norms.   Whole Group 🡪 Sharing   * share/post a big picture plan for our breakout group for the week   Individual/Pair/Whole Group🡪 Graffiti  Where is proportional reasoning found in the grade 11/12 curriculum?   * Participants think individually and record their ideas and then share with a partner. * Chart paper with each course name will be distributed on the tables. Participants will move around the room and record their ideas on the chart paper for each course. On each table there will be “hint” cards which are based on the “Principles and Links” that participants can turn over to help generate additional ideas. * Post and discuss.   Think/Pair/Share 🡪 Problem Solving   * Using BLM 1.1, Connecting Problems to Big Ideas, participants choose one of the problems provided involving from a strand in a Grade 11 or 12 course. * Individually, participants solve the problem, reflect on how the problem is connected to proportional reasoning and match it to the big idea that best reflects it. Scaffolding questions may be used. * Participants should find someone else in the room that solved the same problem and have a standing conversation with them. * Whole group share. | Post Group Norms  Post week at-a-glance  Post Learning goals  Give folders to participants  Probing questions to consider:   * Are the expectations sufficient to get at the big ideas? Why or why not? * What role do students’ prior experiences play in the development of their understanding of the big ideas? * What challenges might you as a teacher face when providing students with learning opportunities that focus on developing understanding of the big ideas? |
| Minds On… |
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|  | Small Group🡪 Matching Activity  Each Big Idea will be recorded on a piece of chart paper and posted in the room.  Each table will have curriculum expectations on strips of paper from a cluster of courses [MEL3E/MBF3C, MEL4E/MAP4C, MCT4C/MHF4U, MCV4U/MDM4U, MCR3U/MCF3M]. Each participant should select a course, then an expectation, think about which Big Idea it connects with, discuss it with a colleague and then post it on the chart paper with that Big Idea. Participants can place as many expectations as time permits.  Participants will receive a copy of all of these expectations which they can use throughout the week.  Whole Group🡪 Gallery Walk  Participants visit all the Big Ideas postings and share and discuss their thinking around the big ideas. | Post mural paper with the big ideas stated  “Were some Big Ideas easier to match than others?”, “Was there anything that stood out for you?” |
| Action! |
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|  | Whole Group 🡪Inside/Outside Circle  Questions reflecting what we’ve learned.  “I think the value of focusing on the same big ideas in proportional reasoning K – 12 might be that.....”  “Did you have any “aha” moments today, or were there ideas that challenged your thinking? If so, what?” |  |
| Consolidate Debrief |
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| Reflection | Home Activity or Further Classroom Consolidation  Refer back to learning goals  Letters from Camppp |  |

BLM 1.1 Connecting Problems to Big Ideas

Complete one of the problems below.

Problem 1

Explain in a variety of ways how you can distinguish the exponential function from the quadratic function and the linear function.

Problem 2

There are two ramps with the following dimensions:

Ramp A – Height is 3m; Base is 4m

Ramp B – Height is 4m; Base is 5m

1. How are the ramps the same? How are they different?
2. Select a Height and Base for Ramp C to make it:
3. as steep as Ramp A
4. steeper than Ramp A, but not as steep as Ramp B
5. twice as steep as Ramp A

Problem 3

Shirley to work on: (MCV4U) Match to BI 5

Problem three: Comparing vector operations to polynomial operations

BLM1.2 Curriculum Expectations

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| **Course** | | **Strand/ Number** | | **Expectation** | | **Big Idea** |
| MEL3E | | EP3.2 | | estimate the sale price before taxes when making a purchase (e.g., estimate 25% off of $38.99 as 25% or ¼ off of $40, giving a discount of about $10 and a sale price of approximately $30; alternatively, estimate the same sale price as about ¾ of $40) | |  |
| MEL3E | | EP3.8 | | compare the unit prices of related items to help determine the best buy  *Sample problem:* Investigate whether or not purchasing larger quantities always results in a lower unit price. | |  |
| MEL3E | | SI2.1 | | determine, through investigation using technology (e.g., calculator, spreadsheet), the effect on simple interest of changes in the principal, interest rate, or time, and solve problems involving applications of simple interest | |  |
| MEL3E | | SI2.4 | | determine, through investigation using technology, the effect on the future value of a compound interest investment of changing the total length of time, the interest rate, or the compounding period  *Sample problem:* Compare the results at age 40 of making a deposit of $1000 at age 20 or a deposit of $2000 at age 30, if both investments pay 6% interest per annum, compounded monthly. | |  |
| MEL3E | | TT2.1 | | determine distances represented on maps (e.g., provincial road map, local street map, web-based maps), using given scales | |  |
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| MBF3C | | MM2.6 | | distinguish exponential relations from linear and quadratic relations by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; inspecting graphs; comparing equations), within the same context when possible (e.g., simple interest and compound interest, population growth) | |  |
| MBF3C | | PF1.6 | | determine, through investigation using technology (e.g., a TVM Solver on a graphing calculator or on a website), the effect on the future value of a compound interest investment or loan of changing the total length of time, the interest rate, or the compounding period ***Sample problem:*** Investigate whether doubling the interest rate will halve the time it takes for an investment to double. | |  |
| MBF3C | | DM2.2 | | determine the theoretical probability of an event (i.e., the ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely), and represent the probability in a variety of ways (e.g., as a fraction, as a percent, as a decimal in the range 0 to 1) | |  |
| MBF3C | | GT1.2 | | represent three-dimensional objects, using concrete materials and design or drawing software, in a variety of ways (e.g., orthographic projections, perspective isometric drawings, scale models) | |  |
| MBF3C | | GT2.1 | | solve problems, including those that arise from real-world applications by determining the measures of the sides and angles of right triangles using primary trigonometric ratios | |  |
| **Course** | | **Strand/ Number** | | **Expectation** | **Big Idea** |
| MCF3M | | EF1.6 | | distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations), within the same context when possible (e.g., simple interest and compound interest, population growth) |  |
| MCF3M | | EF3.1 | | compare, using a table of values and graphs, the simple and compound interest earned for a given principal and a fixed interest rate over time |  |
| MCF3M | | TF1.1 | | solve problems, including those that arise from real-world applications, by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios |  |
| MCF3M | | TF1.3 | | verify, through investigation using technology the sine law and the cosine law |  |
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| MCR3U | | CF3.3 | | simplify rational expressions by adding, subtracting, multiplying, and dividing, and state the restrictions on the variable values |  |
| MCR3U | | EF2.1 | | distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in  a table of values; inspecting graphs; comparing equations) |  |
| MCR3U | | DF2.1 | | identify sequences as arithmetic, geometric, or neither, given a numeric or algebraic representations |  |
| MCR3U | | TF1.1 | | determine the exact values of the sine, cosine, and tangent of the special angles: 0, 30, 45, 60, and 90 |  |
| MCR3U | | TF1.6 | | pose problems involving right triangles and oblique triangles in two dimensional settings, and solve these and other such problems using the primary trigonometric ratios, the cosine law and the sine law |  |
| MCR3U | | TF1.7 | | pose problems involving right triangles and oblique triangles in three dimensional settings, and solve these and other such problems using the primary trigonometric ratios, the cosine law and the sine law |  |

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| **Course** | **Strand/ Number** | **Expectation** | **Big Idea** |
| MEL4E | RD2.1 | determine the theoretical probability of an event (i.e., the ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely), and represent the probability in a variety of ways (e.g., as a fraction, as a percent, as a decimal in the range 0 to 1) |  |
| MEL4E | AM1.4 | convert measures within systems (e.g., centimeters and metres, kilograms and grams, litres and milliliters, feet and inches, ounces and pounds), as required within applications that arise from familiar contexts. |  |
| MEL4E | AM3.1 | identify and describe applications of ratio and rate, and recognize and represent equivalent ratios (e.g., show that 4:6 represents the same ratio as 2:3 by showing that a ramp with a height of 4m and a base of 6m and a ramp with a height of 2m and a base of 3m are equally steep) and equivalent rates (e.g., recognize that paying $1.25 for 250mL of tomato sauce is equivalent to paying $3.75 for 750mL of the same sauce), using a variety of tools |  |
| MEL4E | AM3.2 | identify situations in which it is useful to make comparisons using unit rates, and solve problems that involve comparisons of unit rates |  |
| MEL4E | AM3.3 | identify and describe real-world applications of proportional reasoning (e.g., mixing concrete; calculating dosages; converting units; painting walls; calculating fuel consumption; calculating pay; enlarging patterns), distinguish between a situation involving a proportional relationship and a situation involving a non-proportional relationship in a personal and/or workplace context, and explain their reasoning |  |
| MEL4E | AM3.4 | identify and describe the possible consequences (e.g., overdoses of medication; seized engines; ruined clothing; cracked or crumbling concrete) of errors in proportional reasoning (e.g., not recognizing the importance of maintaining proportionality; not correctly calculating the amount of each component in a mixture) |  |
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| MAP4C | MM2.6 | recognize that a linear model corresponds to a constant increase or decrease over equal intervals and that an exponential model corresponds to a constant *percentage* increase or decrease over equal intervals, select a model (i.e., linear, quadratic, exponential) to represent the relationship between numerical data graphically and algebraically, using a variety of tools (e.g., graphing technology) and strategies (e.g., finite differences,regression), and solve related problems |  |
| MAP4C | MM2.4 | identify when the rate of change is zero, constant or changing, given a table of values or a graph of a relations, and compare two graphs by describing rate of change |  |
| MAP4C | GT1.1 | perform required conversions between the imperial systems and the metric systems using a variety of tools (e.g. tables, calculators, online conversion tools), as necessary within applications |  |
| MAP4C | GT2.1 | solve problems in two dimensions using metric or imperial measurements, including problems that arise from real-world applications by determine the measures of the sides and angles of right triangles using the primary trigonometric ratios, and of acute triangles using the sine law and the cosine law |  |
| MAP4C | DM1.5 | determine an algebraic summary of the relationship between two variables that appear to be linearly related, using a variety of tools and strategies and solve related problems |  |
| MAP4C | DM1.6 | describe possible interpretations of the line of best fit of a scatter plot (e.g. the variables are linearly related) and reasons for misinterpretations (e.g., using too small a sample, failing to consider the effect of outliers; interpolating from a weak correlation; extrapolating non-linearly related data) |  |

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| **Course** | **Strand/ Number** | **Expectation** | **Big Idea** |
| MCT4C | TF1.1 | determine the exact values of the sine, cosine, and tangent of the special angles: 0°, 30°, 45°, 60°, and 90° |  |
| MCT4C | TF1.4 | solve multi-step problems in two and three dimensions, including those that arise from real-world applications (e.g. surveying, navigation), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios |  |
| MCT4C | AG2.2 | perform required conversions between the imperial system and the metric system using a variety of tools as necessary within applications |  |
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| MHF4U | CF1.2 | recognize that the rate of change for a function is a comparison of changes in the dependent variable to changes in the independent variable, and distinguish situations in which the rate of change is zero, constant, or changing by examining applications, including those arising from real-world situations (e.g., rate of change of the area of a circle as the radius increases, inflation rates, the rising trend in graduation rates among Aboriginal youth, speed of a cruising aircraft, speed of a cyclist climbing a hill, infection rates) ***Sample problem:*** The population of bacteria in a sample is 250 000 at 1:00 p.m., 500 000 at 3:00 p.m., and 1 000 000 at 5:00 p.m. Compare methods used to calculate the change in the population and the rate of change in the population between 1:00 p.m. to 5:00 p.m. Is the rate of change constant? Explain your reasoning. |  |
| MHF4U | CF3.1 | compare, through investigation using a variety of tools and strategies the characteristics of various functions |  |
| MHF4U | TF1.1 | recognize the radian as an alternative unit to the degree for angle measurement, define the radian measure of an angle as the length of the arc that subtends this angle at the centre of a unit circle, and develop and apply the relationship between radian and degree measure |  |
| MHF4U | TF1.4 | determine, without technology, the exact values of the primary trigonometric ratios and the reciprocal trigonometric ratios for the special angles and their multiples less than or equal to 2π |  |
| MFH4U | TF3.3 | recognize that trigonometric identities are equations that are true for every value in the domain (i.e., a counter-example can be used to show that an equation is not an identity), prove trigonometric identities through the application of reasoning skills, using a variety of relationships (e.g., tan x = sin x/cos x; sin2x + cos2x = 1; the reciprocal identities; the compound angle formulas), and verify identities using technology  Sample problem: Use the compound angle formulas to prove the double angle formulas. |  |

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| **Course** | **Strand/ Number** | **Expectation** | **Big Idea** |
| MDM4U | CP1.3 | determine the theoretical probability, Pi of each outcome of a discrete sample space, recognize that the sum of the probabilities of the outcomes is 1, recognize that the probabilities Pi form the probability distribution associated with the sample space, and solve related problems |  |
| MDM4U | CP2.2 | solve simple problems using techniques for counting permutations and combinations, where all objects are distinct, and express the solutions using standard combinatorial notation [e.g., n!, P(n, r), (n r)]  Sample problem: In many Aboriginal communities, it is common practice for people to shake hands when they gather. Use combinations to determine the total number of handshakes when 7 people gather, and verify using a different strategy. |  |
| MDM4U | CP2.3 | solve introductory counting problems involving the additive counting principle (e.g., determining the number of ways of selecting 2 boys or 2 girls from a group of 4 boys and 5 girls) and the multiplicative counting principle (e.g., determining the number of ways of selecting 2 boys and 2 girls from a group of 4 boys and 5 girls) |  |
| MDM4U | SA3.1 | interpret statistics presented in the media (e.g., the UN's finding that 2% of the world's population has more than half the world's wealth, whereas half the world's population has only 1% of the world's wealth), and explain how the media, the advertising industry, and others (e.g., marketers, pollsters) use and misuse statistics (e.g., as represented in graphs) to promote a certain point of view (e.g., by making a general statement based on a weak correlation or an assumed cause-and-effect relationship; by starting the vertical scale at a value other than zero; by making statements using general population statistics without reference to data specific to minority groups) |  |
| MDM4U | SA3.2 | assess the validity of conclusions presented in the media by examining sources of data, including Internet sources, methods of data collection, and possible sources of bias, and by questioning the analysis of the data and conclusions drawn from the data |  |
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| MCV4U | RC2.5 | determine, through investigation using technology, the graph of the derivative f'(x)or dy/dx of a given exponential function [i.e., f(x) = ax > 0, a ≠ 1)] [e.g., by generating a table of values showing the instantaneous rate of change of the function for various values of x and graphing the ordered pairs; by using dynamic geometry software to verify that when f(x) = ax, f'(x) = kf(x)], and make connections between the graphs of f(x) and f'(x) or y and dy/dx [e.g., f(x) and f'(x) are both exponential; the ratio f'(x)/f(x) is constant, or f'(x) = kf(x); f'(x) is a vertical stretch from the x-axis of f(x)]  Sample problem: Graph, with technology, f(x) = a(x) (a > 0, a≠ 1) and f'(x) on the same set of axes for various values of a (e.g., 1.7, 2.0, 2.3, 3.0, 3.5). For each value of a, investigate the ratio f'(x)/f(x) for various values of x, and explain how you can use this ratio to determine the slopes of tangents to f(x). |  |
| MCV4U | RC2.8 | verify, using technology (e.g., calculator, graphing technology), that the derivative of the exponential function f(x) = ax is f'(x) = axln a for various values of *a* [e.g., verifying numerically for f(x) = 2x that f'(x) = 2x ln 2 by using a calculator to show that is ln 2 or by graphing f(x) = 2x, determining the value of the slope and the value of the function for specific x-values, and comparing the ratio f'(x)/f(x) with ln 2] Sample problem: Given f(x) = ex, verify numerically with technology using that f'(x) = f(x)ln e. |  |
| MCV4U | GA1.2 | represent a vector in two-space geometrically as a directed line segment, with directions expressed in different ways, and algebraically, and recognize vectors with the same magnitude and direction but different positions as equal vectors |  |
| MCV4U | GA2.1 | perform the operations of addition, subtraction, and scalar multiplication on vectors represented as directed line segments in two-space, and on vectors represented in Cartesian form in two-space and three-space |  |